SUPPLEMENTARY PROBLEMS FOR CHAPTER 4

1. State which of the following represent legitimate correlation functions, and if not tell why not.

   (a) $R_x[l] = 1.5 \quad \forall l$

   (b) $R_x[l] = \dfrac{\sin l}{l}$

   (c) $R_x[l] = (\sin l)^2$

   (d) $R_x[l] = e^{-l} \sin l$

   (e) $R_x[l] = 1 - u[l-4] - u[-4-l]$

   (f) $R_x[l] = \begin{cases} \cos l & -\pi < l < \pi \\ 0 & \text{otherwise} \end{cases}$

   (g) $R_x[l] = \dfrac{1}{|l|!}$

2. State which of the following represent legitimate power spectral density functions for a discrete random process, and if not tell why not.

   (a) $S_x(e^{\jmath\omega}) = e^{-\frac{\omega^2}{2}}$

   (b) $S_x(e^{\jmath\omega}) = e^{\sin \omega}$

   (c) $S_x(e^{\jmath\omega}) = \sqrt{\cos \omega}$

   (d) $S_x(e^{\jmath\omega}) = 0.2 + 0.8(1 + \cos \omega)$

   (e) $S_x(e^{\jmath\omega}) = \dfrac{\sin \omega}{\omega^3}$

   (f) $S_x(e^{\jmath\omega}) = \begin{cases} 1 & -\frac{\pi}{2} + 2\pi k < \omega < \frac{\pi}{2} + 2\pi k \\ 0 & \text{otherwise} \end{cases}$

   (g) $S_x(e^{\jmath\omega}) = \delta_c(e^{\jmath\omega} - 1)$

3. A random process is defined by

$$x[n] = e^{\jmath(\omega_1 n + \phi_1)} + 2e^{\jmath(\omega_2 n + \phi_2)} + u[n] + v[n]$$

   where ω_1 and ω_2 are known frequencies, ϕ_1 and ϕ_2 are two independent random variables uniformly distributed between $-\pi$ and π, and $u[n]$ and $v[n]$ are two mutually independent sequences of zero mean, independent identically distributed random variables with variances σ_u^2 and σ_v^2. The random sequences u and v are also independent of the random variables ϕ_1 and ϕ_2.

(a) Compute the correlation function $\mathrm{R}_x[n_1, n_0]$.

(b) Is $x[n]$ (wide sense) stationary?

4. Tell if the following could represent legitimate quantities for a random process.

(a)

$$R_x[l] = |l|e^{-|l|}$$

(b)

$$R_x[l] = 10e^{-2|l|} + 5e^{-3|l|} + 2\delta[l]$$

(c)

$$R_x[l] = \frac{l^2 + 3}{l^2 + 6}$$

(d)

$$S_x(e^{\jmath\omega}) = \left|\frac{1}{1 - e^{-\jmath\omega}}\right|^2$$

(e)

$$S_x(e^{\jmath\omega}) = \begin{cases} 1 - \frac{\omega}{\pi} & 0 \le \omega < \pi \\ 0 & -\pi \le \omega < 0 \end{cases}$$

(Assume this repeats periodically.)

(f)

$$S_x(z) = \frac{z^2 - 1}{z^2 + 1}$$

(g)

$$S_x(z) = \frac{8}{2z - 5 + 2z^{-1}}$$

(h)

$$S_x(z) = z + z^{-1}$$

5. $x[n]$ and $y[n]$ are two jointly stationary random processes. Tell if $R[l]$ as defined by each of the following expressions could be *guaranteed* to be a legitimate correlation function.

   (a) $R[l] = R_x[l] + R_y[l]$

   (b) $R[l] = R_x[l] - R_y[l]$

   (c) $R[l] = R_{xy}[l] + R_{yx}[l]$

   (d) $R[l] = R_x[l] + R_{xy}[l] + R_{yx}[l]$

   (e) $R[l] = R_x[l] + R_y[l] + R_{xy}[l] + R_{yx}[l]$

6. Find the correlation function and power spectral density function for the random square wave of Prob. 4.24 (c) [text p. 220].

   Hint: Assume that P is an even integer. Write the square wave in a discrete Fourier series and apply the results of Table 4.6 [text p. 185].

7. A discrete random process consists of independent identically-distributed random variables $x[n]$ described by

$$\Pr\left[x[n] = r\right] = \begin{cases} \left(\frac{1}{2}\right)^{(r+1)} & r = 0, 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

   A related random process $y[n]$ is defined as the difference

$$y[n] = x[n] - x[n-1]$$

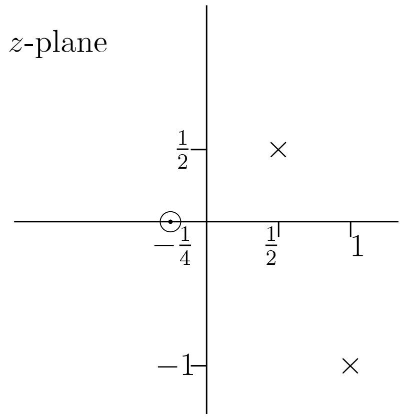
   (a) What are the mean and variance of $x[n]$?

   (b) Is $y[n]$ stationary?

   (c) What are the mean and variance of $y[n]$?

   (d) Find and sketch the correlation function of $y[n]$.

8. Some of the poles and zeros of a complex spectral density function $S_x(z)$ are shown below. Draw the remaining poles and zeros and label their positions in the complex plane. Does $S_x(z)$ correspond to a *real* random process? Find the complex spectral density function to within a scale factor.



9. Find the correlation function corresponding to the following complex spectral density function:
$$S_x(z) = \frac{2z - 8 + 2z^{-1}}{2z - 5 + 2z^{-1}}$$

10. A random process is defined by
$$y[n] = \sum_{k=1}^{n} x[k]$$
where $x[n]$ is a Bernoulli process with parameter P. Compute the mean, correlation function, and covariance function for $y[n]$. Is $y[n]$ stationary?

11. Consider the two-state Markov chain described on page 105 in the text (Fig. 3.9 and Table 3.3). Let the values of the states be $\mathrm{S}_1 = +1$ and $\mathrm{S}_2 = -1$. Assume the process has been running for a long time. Find and sketch the correlation function for the Markov process for lag values $l = 0, 1, 2,$ and 3.

12. Tell which of the following are legitimate complex power spectral density functions and why or why not.

$$\text{(i)} \quad S_x(z) = z + z^{-1} + \tfrac{1}{4}$$

$$\text{(ii)} \quad S_x(z) = \frac{4z}{-2z^2 + 5z - 2}$$

13. Tell if the following are legitimate and why or why not.

    (a) $R_x[l] = e^{-l}\cos l$

    (b) $S_x(e^{\jmath\omega}) = e^{\jmath\omega} + e^{-\jmath\omega}$

    (c) $S_x(z) = \dfrac{z^2 - 1}{z^2 - 5z + 6}$

14. Find the correlation function $R_x[l]$ corresponding to the following complex spectral density function. Use any method.

$$S_x(z) = \frac{7}{-12z + 25 - 12z^{-1}}$$